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Brief communication

Comments on "A turbulent diffusion model for particle dispersion and deposition in horizontal tube flow"

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1. Introduction

An error was found in the paper "A Turbulent Diffusion Model for Particle Dispersion and Deposition in Horizontal Tube Flow" by Mols and Oliemans (1998). The error affected the results presented in the first part of the paper, namely, Figs. 6–11. The objective of this Brief communication is to correct the error providing the proper solution, and to present the main results with the corrected solution. Note that the nomenclature used is the same as in Mols and Oliemans (1998).

2. Correction for analytical solution

The physical phenomena of particle deposition in turbulent annular flow in horizontal pipes were presented by Binder and Hanratty (1992). Mols and Oliemans (1998) extended the study and also presented the mathematical formulation of the problem. The final analytical solution presented by Mols and Oliemans (1998) is given in

$$C^+(y^+,t^+) = \exp(ay^+)\sum_{n=0}^{\infty}\gamma_n[\cos(b_n y^+) + \beta_n \sin(b_n y^+)]\exp(-k_n^2 D_p^+ t^+), \tag{1}$$

where b_n, then satisfies the transcendental equation,

$$\tan(b_n) = \frac{2\lambda b_n}{(a^2-\lambda^2)+b_n^2}. \tag{2}$$

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In (2), λ is the dimensionless free-flight/diffusion ratio and a is equal to $-P/2$, where P is the Peclet number. The eigenvalues k_n were defined in terms of b_n and a

$$k_n^2 = b_n^2 + a^2, \tag{3}$$

and

$$\beta_n = \frac{\lambda - a}{b_n}. \tag{4}$$

In order to satisfy the initial condition, Mols and Oliemans (1998) define γ_n as:

$$\gamma_n = \frac{\int_0^1 C^+(t^+=0)\,\exp(ay^+)[\cos(b_n y^+) + \beta_n \sin(b_n y^+)]\,\mathrm{d}y^+}{\int_0^1 \exp(2ay^+)[\cos(b_n y^+) + \beta_n \sin(b_n y^+)]^2\,\mathrm{d}y^+}. \tag{5}$$

An error occurred in (5). In order to demonstrate the error and the correct solution, γ_n is recalculated from (1) applying the initial condition $C^+(y^+,0) = C^+(t^+=0)$, as follows:

$$C^+(t^+=0) = \exp(ay^+)\sum_{n=0}^{\infty} \gamma_n[\cos(b_n y^+) + \beta_n \sin(b_n y^+)]. \tag{6}$$

The general Fourier coefficient γ_n must be solved from (6). The eigenfunctions of any regular Strum–Liouville eigenvalue problem are always orthogonal relative to a weight function $\sigma(y^+)$. The weight function $\sigma(y^+)$ can be calculated from the original diffusion equation as proposed by Mols and Oliemans (1998). Applying separation of variables method to the diffusion equation results in two ODEs. The ODE for the space variable y^+ is given by

$$\frac{\mathrm{d}^2F(y^+)}{\mathrm{d}y^{+^2}} - 2a\frac{\mathrm{d}F(y^+)}{\mathrm{d}y^+} + k^2F(y^+) = 0. \tag{7}$$

The weight function $\sigma(y^+)$ is calculated from Eq. (7) using an "integrating factor" type technique (Constanda, 2002), as given by

$$\sigma(y^+) = \exp\left(\int -2a\,\mathrm{d}y^+\right) = \exp(-2ay^+). \tag{8}$$

In order to solve for γ_n, the orthogonality of the following expression must be verified:

$$\int_0^1 [\cos(b_n y^+) + \beta_n \sin(b_n y^+)][\cos(b_m y^+) + \beta_m \sin(b_m y^+)]\,\mathrm{d}y^+ \begin{matrix} = 0 & \text{if } n \neq m \\ \neq 0 & \text{if } n = m \end{matrix}. \tag{9}$$

Eq. (2) was solved numerically for values of n and m in the interval $(1,1000)$, where $n \neq m$, and the results substituted into (9). All the residuals from the solution of (9) were below 10^{-6}. Then, the correct solution for the eigenvalue γ_n is given in (10), which should replace the erroneous expression in (5).

$$\gamma_n = \frac{\int_0^1 C^+(t^+=0)\,\exp(-ay^+)[\cos(b_n y^+) + \beta_n \sin(b_n y^+)]\,\mathrm{d}y^+}{\int_0^1 [\cos(b_n y^+) + \beta_n \sin(b_n y^+)]^2\,\mathrm{d}y^+}. \tag{10}$$

3. Results and discussion

Tables 4 and 5 in Mols and Oliemans (1998) give the properties of the fluid and particles that were used to generate their results. The same properties are used in this study. Note that in the results presented here, it is assumed that droplets travel axially at a constant velocity. Thus, the distance traveled downstream can be easily determined by the product of the time and the gas velocity.

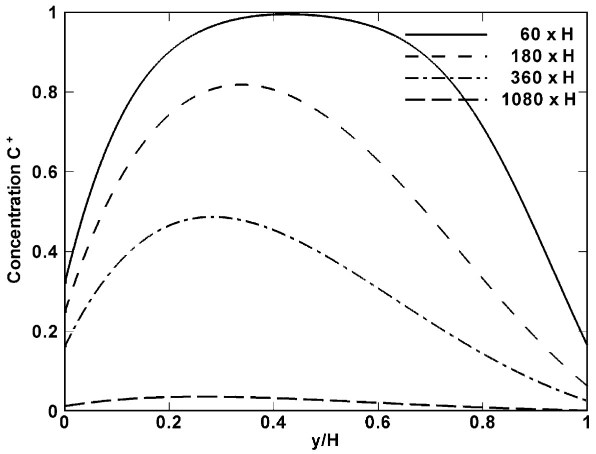

Fig. 1. Concentration profiles for 50 μm particles and Fr* = 14.9; uniform initial distribution.

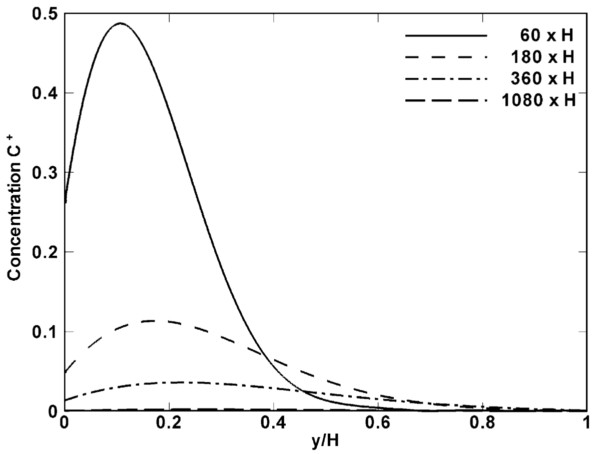

Fig. 2. Concentration profiles for 50 μm particles and Fr* = 14.9; initial delta source at the bottom.

Fig. 1 replaces Fig. 6 in the original paper, presenting the one-dimensional concentration profiles, of a uniform initial concentration, for 50 μm particles and Fr* = 14.9 at different distances traveled downstream. As can be seen in Fig. 1, gradual deposition of droplets along the channel is predicted. The maximum concentration location shifts towards the bottom wall downstream from the inlet, while the concentration in the core of the flow decreases. Fig. 6 in the original paper, on the other hand, exhibits fast droplet deposition for a short distance of 1.24 H, while the maximum concentration location moves upward against gravity along the channel, which is believed to be the wrong trend.

Fig. 2 (replacing Fig. 8 in the original paper) show the one-dimensional concentration profiles, of an initial delta source located at the bottom of the channel, for 50 μm particles and Fr* = 14.9 at different distances traveled downstream. Comparing the current Fig. 2 to the corresponding original Fig. 8, although the curves in both figures look similar in shape, the distances from the inlet associated with the curves are different. For example, the distance required for full deposition in the original figure is very short, namely $250 \times H$, while the present solution with the proper mathematical expression for γ_n predicts a distance of $1000 \times H$.

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